1 Gaussian Elimination

1.1 Concepts

1. In order to solve a system of equations to find the solution or determine if there are zero or infinitely many solutions, use Gaussian elimination on the **augmented matrix**, a matrix formed by appending the answer vector to the original matrix. A system of equations is **consistent** if there is at least one solution and **inconsistent** if there are no solutions.

1.2 Problems

2. True   False   As soon as we see a row like $(000\ldots0|0)$ during Gaussian elimination, we know that the system will have infinitely many solutions.

3. True   False   If we see a row like $(000\ldots0|0)$ then we know the determinant of the matrix.

4. Use Gaussian elimination on the following augmented matrix. Write the equations these correspond to.
$$\left(\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 0 & -1 & -1 & 2 \\ -3 & 0 & -2 & -1 \end{array}\right)$$

5. Use Gaussian elimination to solve the following system of equations:
$$\begin{cases} 2x_1 + x_2 - x_3 = 4 \\ -4x_1 - 2x_2 + 2x_3 = -6 \\ 6x_1 + 3x_2 - 3x_3 = 12 \end{cases}$$

6. Find conditions on a, b such that the following system has no solutions, infinitely many, and a unique solution.
$$\begin{cases} x + ay = 2 \\ 4x + 8y = b \end{cases}$$

7. Find $\begin{pmatrix} 1 & 3 & 1 \\ 0 & 1 & 1 \\ 2 & 5 & 2 \end{pmatrix}^{-1}$.

8. Use Gaussian elimination to solve the following system of equations:

$$\begin{cases} x_1 - 2x_2 - 6x_3 = 5 \\ 2x_1 + 4x_2 + 12x_3 = -6 \\ x_1 - 4x_2 - 12x_3 = 9 \end{cases}$$

9. Come up with an example of a consistent system of equations with 3 equations and 2 variables. Give an example of an inconsistent system of linear equations with 2 equations and 3 variables.

10. Use Gaussian elimination to solve the following system of equations:

$$\begin{cases} z - 3y = -6 \\ x - 2y - 2z = -14 \\ 4y - x - 3z = 5 \end{cases}$$